Int. J. Solids Structures, 1969, Vol. 5, pp. 1107 to 1117. Pergamon Press. Printed in Great Britain

# STABILITY OF A THICK RUBBER SOLID SUBJECT TO PRESSURE LOADS

CHIEN-HENG WU and OTTO E. WIDERA

Department of Materials Engineering, University of Illinois at Chicago Circle, Chicago, Illinois

**Abstract**—The stability of a rectangular solid in plane strain subjected to a constant axial and lateral pressure is investigated. Two types of lateral pressure are considered, namely, a hydrostatic pressure and a constant directional pressure. Problems of this type have been studied by Kerr and Tang for a perfectly elastic material of the harmonic type. Constitutive equations of this type do not, however, give a good description of known materials unless the strains are not too large. They have found that, for the case of antisymmetric deformation (bending), the constant directional pressure destabilizes the solid while the hydrostatic pressure stabilizes the solid. Moreover the solid is unstable for both cases when the lateral pressure is equal to the axial pressure. In this paper, a Mooney material is considered. It is found that both types of lateral pressure stabilize the solid for both antisymmetric and symmetric deformations.

## 1. INTRODUCTION

THE instability of a rectangular solid in plane strain subjected axially to a constant pressure* $Cp_1$ and laterally to a constant pressure $Cp_2$, Fig. 1, has been studied in a series of papers by Kerr and Tang [1–3]. A fairly complete list of related references has been given in [3]. Two types of lateral pressure have been considered, a hydrostatic pressure and a constant directional pressure. They have found that the constant directional $p_2$ destabilizes the solid and the hydrostatic $p_2$ stabilizes the solid for $p_2$ less than approximately $2p_1$. Moreover, the solid is unstable when $p_1 = p_2$.

The constitutive relation used by Kerr and Tang was derived from the so called "standard" strain energy density. Materials with a standard strain energy density function are called harmonic materials and have been treated by John [4]. While the standard strain energy density function has the advantage that the corresponding solution reduces to that of the linear elasticity equations for infinitesimal deformations, it does not give a good description of known materials unless the strains are not too large. This deficiency has been demonstrated by Sensenig [5].

The problem studied by Kerr and Tang involves very large deformations if the solid is not too thin or the ratio $p_2/p_1$ is not too small. It is felt, therefore, that their results may not even serve as a qualitative description of the true behavior of a real material for large deformations. To examine the effect of large deformations we have solved the same problem by using the general nonlinear theory given in Green and Zerna [6]. Unfortunately, no simple constitutive relation is available for compressible materials and, therefore, no direct comparison can be made. Nevertheless our results are valid for large deformations and are completely different from those obtained by Kerr and Tang.

We have solved the problem for a Mooney solid and found that both the constant directional and the hydrostatic $p_2$ stabilizes the solid. While the stabilizing effect due to a

* $C$ is a material constant involved in the analysis.

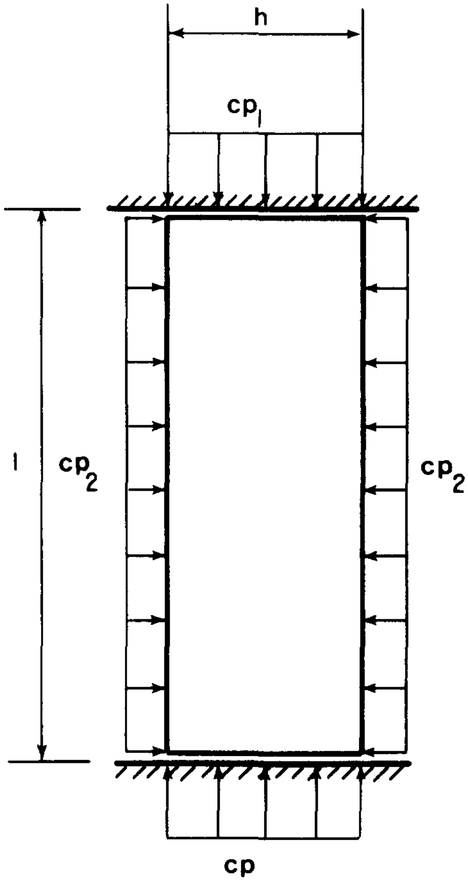


FIG. 1.

hydrostatic $p_2$ is independent of the thickness of the solid, the stabilizing effect due to a constant directional $p_2$ is stronger for a thinner solid. Furthermore, the solid is stable when $p_1 = p_2$. These results hold for both antisymmetric instability (bending) and symmetric instability (bulging). The case of symmetric instability has not been treated numerically by Kerr and Tang due to the obvious reason that very large strains are involved.

Finally, it should be mentioned that the stability of Mooney type solids of other geometries has been studied by Rivlin, Wilkes and others.*

## 2. SMALL DEFORMATION SUPERPOSED ON FINITE UNIFORM EXTENSIONS IN TWO PERPENDICULAR DIRECTIONS†

Let us consider a rectangular solid $B_0$ occupying the region‡ $0 \leq x_1 \leq 1$, $-h/2 \leq x_2 \leq h/2$, $-\infty \leq x_3 \leq +\infty$, where $x_i$ is a rectangular system. The body $B_0$ is deformed into a body $B$ by two uniform finite extensions (compressions) along the $x_1$ and

* A list of references may be found in [6, 7].

† This section is an abridged version of the results given in Green and Zerna [6]. The reader is referred to [6] for details of the notation and the derivation.

‡ The coordinates have been nondimensionalized.

$x_2$ directions. We take a set of moving coordinates $\theta_i$ to coincide with a fixed rectangular system of coordinates $(x, y, z)$ in the strained body $B$. Thus we have

$$\theta_1 = x, \qquad \theta_2 = y, \qquad \theta_3 = z \tag{2.1}$$

If the axes $(x, y, z)$ are taken to coincide with the rectangular axes then

$$x_1 = \frac{x}{\lambda_1}, \qquad x_2 = \frac{y}{\lambda_2}, \qquad x_3 = \frac{z}{\lambda_3} \tag{2.2}$$

where $\lambda_1$, $\lambda_2$ and $\lambda_3 = 1$ are the constant extension ratios. If the material is incompressible, then $\lambda_1 = 1/\lambda_2$ since $\lambda_1\lambda_2\lambda_3 = 1$ and $\lambda_3 = 1$. Equation (2.2) may now be written as

$$x_1 = \frac{x}{\lambda}, \qquad x_2 = y\lambda, \qquad x_3 = z \tag{2.3}$$

where $\lambda \equiv \lambda_1 = 1/\lambda_2$.

It follows from (2.3) and (2.1) that the covariant metric tensors $g_{ij}$ and $G_{ij}$ of $B_0$ and $B$, respectively, are

$$g_{ij} = \begin{bmatrix} 1/\lambda^2 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad G_{ij} = \delta_{ij} \tag{2.4}$$

where $\delta_{ij}$ is the Kronecker delta. The strain invariants associated with body $B$ are

$$\begin{aligned} I_1 &= \lambda_1^2+\lambda_2^2+\lambda_3^2 = \lambda^2+\frac{1}{\lambda^2}+1 \\ I_2 &= \lambda_1^{-2}+\lambda_2^{-2}+\lambda_3^{-2} = \frac{1}{\lambda^2}+\lambda_2+1 \\ I_3 &= 1. \end{aligned} \tag{2.5}$$

If the strain-energy function $W$ is of the form

$$W = C[(I_1-3)+K(I_2-3)] \tag{2.6}$$

where $C$ and $K$ are constants, then the stress components $\tau^{ij}$ for body $B$ are, apart from the constant $C$,

$$\tau^{ij} = 2g^{ij}+2KB^{ij}+p\,\delta^{ij}. \tag{2.7}$$

Here, tensor $B^{ij}$ is given by

$$B^{ij} = I_1 g^{ij} - g^{ik}g^{jk} \tag{2.8}$$

and $Cp$ is a hydrostatic pressure. It follows from (2.7, 2.8) that

$$\begin{aligned} \tau^{11} &= 2(1+K)\lambda^2+2K+p \\ \tau^{22} &= 2(1+K)\frac{1}{\lambda^2}+2K+p \\ \tau^{33} &= 2+2K\left(\lambda^2+\frac{1}{\lambda^2}\right)+p \\ \tau^{12} &= \tau^{13} = \tau^{23} = 0. \end{aligned} \tag{2.9}$$

Since $\lambda$ and $p$ are constants for the present problem, the stress equations of equilibrium are satisfied when the body forces are zero. If body $B$ is held in equilibrium by two constant pressures $Cp_1$ and $Cp_2$, then we must set

$$\begin{aligned} 2(1+K)\lambda^2+2K+p &= -p_1 \\ 2(1+K)\frac{1}{\lambda^2}+2K+p &= -p_2. \end{aligned} \tag{2.10}$$

Equation (2.10) can be solved for the two unknown constants $\lambda$ and $p$.

To study the stability of the strained body $B$ we consider a slightly buckled body $B'$. A point $P$ in body $B$ is carried to a point $P'$ in body $B'$ by the dimensionless displacements $w_\alpha(\theta_1, \theta_2)$, $\alpha = 1$ and 2, where $w_\alpha$ are the covariant components of the displacement vector $\mathbf{w}$ referred to the base vectors at point $P$ of body $B$.

The covariant metric tensor of body $B'$ is

$$G_{ij}+G'_{ij} = \delta_{ij}+(w_{i,j}+w_{j,i}) \tag{2.11}$$

where $w_{i,j} = \partial w_i/\partial\theta^j$. The strain invariants associated with body $B'$ are

$$\begin{aligned} I_1+I'_1 &= I_1+g^{\alpha\beta}G'_{\alpha\beta} \\ I_2+I'_2 &= I_2+g^{\alpha\beta}G'_{\alpha\beta} \\ I_3+I'_3 &= I_3+\delta^{\alpha\beta}G'_{\alpha\beta}. \end{aligned} \tag{2.12}$$

For the strained body $B'$ the strain energy function corresponding to (2.6) becomes

$$W = C_1[(I_1+I'_1-3)+K(I_2+I'_2-3)]. \tag{2.13}$$

The corresponding stress tensor for the strained body $B'$ is $\tau^{ij}+\tau'^{ij}$ where

$$\tau'^{ij} = 2KB'^{ij}+pG'^{ij}+p'\delta^{ij}. \tag{2.14}$$

Here, tensor $B'^{ij}$ is given by

$$B'^{ij} = (g^{ij}g^{\alpha\beta}-g^{i\alpha}g^{j\beta})G'_{\alpha\beta} \tag{2.15}$$

and $C_1p'$ is again a hydrostatic pressure. It follows from (2.14, 2.15) that

$$\begin{aligned} \tau'^{11} &= 4Kw_{2,2}-2pw_{1,1}+p' \\ \tau'^{22} &= 4Kw_{1,1}-2pw_{2,2}+p' \\ \tau'^{33} &= 4K\left(\frac{1}{\lambda^2}w_{1,1}+\lambda^2 w_{2,2}\right)+p' \\ \tau'^{12} &= -(2K+p)(w_{1,2}+w_{2,1}) \\ \tau'^{13} &= \tau'^{23} = 0. \end{aligned} \tag{2.16}$$

The equations of equilibrium reduce to

$$(\tau'^{ij}+\tau^{ik}w_{j,k}+\tau^{kj}w_{i,k})_{ji} = 0. \tag{2.17}$$

Substitution of (2.16) into (2.17) yields

$$(\tau^{11}-2K-p)w_{1,11}+(\tau^{22}-2K-p)w_{1,22}+p'_{,1} = 0$$
$$(\tau^{11}-2K-p)w_{2,11}+(\tau^{22}-2K-p)w_{2,22}+p'_{,2} = 0. \tag{2.18}$$

The third equation of equilibrium is satisfied identically. To obtain a third equation for the determination of the three unknown functions $w_1$, $w_2$ and $p'$, we use the condition

$$I'_3 = 0. \tag{2.19}$$

Equations (2.19, 2.12) imply that

$$w_{1,1}+w_{2,2} = 0. \tag{2.20}$$

On substituting (2.9) into (2.18), we obtain

$$2(1+K)\lambda^2 w_{1,11}+2(1+K)\frac{1}{\lambda^2}w_{1,22}+p'_{,1} = 0$$
$$2(1+K)\lambda^2 w_{2,11}+2(1+K)\frac{1}{\lambda^2}w_{2,22}+p'_{,2} = 0. \tag{2.21}$$

Equations (2.20, 2.21) together with certain homogeneous boundary conditions constitute an eigenvalue problem for the determination of $\lambda$. This, in turn, determines the ratio of $p_1$ and $p_2$ from equation (2.10).

## 3. BOUNDARY CONDITIONS

Let $y_i$ be a set of moving rectangular cartesian coordinates in the body $B'$ and let the axes $(y_1, y_2, y_3)$ coincide with the fixed axes $(x, y, z)$. Then

$$y_i = \theta_i + w_i. \tag{3.1}$$

We shall denote the base vectors at $P$ in the body $B$ by $\mathbf{G}^i$ and the base vectors at $P'$ in the body $B'$ by $\mathbf{G}^i+\mathbf{G}'^i$.

The surfaces $x_2 = \pm h/2$ are parametrized by the equations

$$F(\theta_1, \theta_2, \theta_3) = \theta_2 \pm \frac{h}{2\lambda} = 0. \tag{3.2}$$

Referred to $y_i$-axes, the unit normal to (3.2) has directions proportional to $\partial F/\partial y_i$. The unit normal $\mathbf{n}$ to the surface (3.2) may be written as

$$\mathbf{n} = (n_i+n'_i)(\mathbf{G}^i+\mathbf{G}'^i). \tag{3.3}$$

It follows from a simple tensor transformation that

$$n_i+n'_i = J\frac{\partial F}{\partial y_j}\frac{\partial y_j}{\partial \theta^i} = J\frac{\partial F}{\partial \theta^i} \tag{3.4}$$

where

$$J = \left[\frac{\partial F}{\partial \theta^i}\frac{\partial F}{\partial \theta^j}(G^{ij}+G'^{ij})\right]^{-\frac{1}{2}} = 1+w_{2,2} \tag{3.5}$$

is a normalizing constant.

The contravariant components $C(t^i+t'^i)$ of the surface traction on the surface (3.2) of the body $B'$ are

$$\begin{aligned} t^i+t'^i &= (\tau^{ji}+\tau'^{ji})(n_j+n'_j) \\ &= \tau^{2i}+(\tau'^{2i}+\tau^{2i}w_{2,2}). \end{aligned}$$

The physical components $C(T^i+T'^i)$ of this traction are then given by

$$\begin{aligned} T^1+T'^1 &= \tau^{21}+[\tau'^{21}+\tau^{21}(w_{1,1}+w_{2,2})] \\ T^2+T'^2 &= \tau^{22}+[\tau'^{22}+2\tau^{22}w_{2,2}]. \end{aligned} \tag{3.6}$$

If the surface traction is a hydrostatic pressure $p_2$, then

$$\tau^{21} = 0, \qquad \tau^{22} = -p_2 \tag{3.7}$$

$$\tau'^{21} = 0 \tag{3.8}$$

$$\tau'^{22}-2p_2w_{2,2} = 0. \tag{3.9}$$

The second condition of (3.7) is just the second of (2.10).

Alternatively, we can refer the components of surface traction to the $y_i$-axes. If these components are denoted by $s^i+s'^i$ then

$$s^i+s'^i = (t^j+t'^j)\frac{\partial y_i}{\partial \theta_j} = (t^j+t'^j)\left(\delta^i_j+\frac{\partial w_i}{\partial \theta_j}\right). \tag{3.10}$$

These components are also the physical components $S^i+S'^i$ of stress referred to $y_i$-axes. Thus

$$\begin{aligned} S^1+S'^1 &= \tau^{21}+[\tau'^{21}+\tau^{22}w_{1,2}+\tau^{21}(w_{1,1}+w_{2,2})] \\ S^2+S'^2 &= \tau^{22}+[\tau'^{22}+2\tau^{22}w_{2,2}+\tau^{21}w_{2,1}]. \end{aligned} \tag{3.11}$$

If the surface traction on (3.2) is a constant directional pressure $p_2$ parallel to the fixed $y$-axis, then

$$\tau^{21} = 0, \qquad \tau^{22} = -p_2 \tag{3.12}$$

$$\tau'^{21}-p_2w_{1,2} = 0 \tag{3.13}$$

$$\tau'^{22}-2p_2w_{2,2} = 0. \tag{3.14}$$

We note that (3.13) is different from (3.8).

The boundary conditions on the surfaces $x_1 = 0, 1$ are taken to be

$$\tau^{11} = -p_1, \qquad \tau^{12} = 0 \tag{3.15}$$

$$w_1 = 0, \qquad \tau'^{12} = 0. \tag{3.16}$$

Substituting (2.16) into (3.8, 3.9, 3.13, 3.14, 3.16) and applying (2.10, 2.20) we obtain the explicit boundary conditions for the following two cases:

*Case I: Hydrostatic pressure* $p_2$

$$w_1 = 0, \qquad w_{2,1} = 0 \quad \text{at } \theta_1 = 0, \lambda \tag{3.17}$$

$$w_{1,2}+w_{2,1} = 0 \quad \text{at } \theta_2 = \pm\frac{h}{2\lambda} \tag{3.18}$$

$$4(1+K)\frac{1}{\lambda^2}w_{2,2}+p' = 0 \quad \text{at } \theta_2 = \pm\frac{h}{2\lambda}\cdot \tag{3.19}$$

*Case II: Constant directional* $p_2$

$$w_1 = 0, \qquad w_{2,1} = 0 \quad \text{at } \theta_1 = 0, \lambda \tag{3.20}$$

$$2(1+K)\frac{1}{\lambda^2}w_{1,2}+\left[p_2+2(1+K)\frac{1}{\lambda^2}\right]w_{2,1} = 0 \quad \text{at } \theta_2 = \pm\frac{h}{2\lambda} \tag{3.21}$$

$$4(1+K)\frac{1}{\lambda^2}w_{2,2}+p' = 0 \quad \text{at } \theta_2 = \pm\frac{h}{2\lambda}\cdot \tag{3.22}$$

## 4. SOLUTION

We shall first obtain a set of solutions satisfying the differential equations (2.20, 2.21) and the boundary conditions at $\theta_1 = 0, \lambda$. It may be verified that these equations and conditions are satisfied by

$$\begin{aligned} w_1(\theta_1,\theta_2) &= -\frac{1}{w_n}\frac{\mathrm{d}f(\theta_2)}{\mathrm{d}\theta_2}\sin\omega_n\theta_1 \\ w_2(\theta_1,\theta_2) &= f(\theta_2)\cos\omega_n\theta_1 \\ p^1(\theta_1,\theta_2) &= g(\theta_2)\cos\omega_n\theta_1 \end{aligned} \tag{4.1}$$

where

$$\omega_n = \frac{n\pi}{\lambda}, \qquad n = 1, 2, 3, \ldots \tag{4.2}$$

$$\begin{aligned} f(\theta_2) = {} & A\cosh\omega_n\theta_2+B\cosh\lambda^2\omega_n\theta_2 \\ & +C\sinh\omega_n\theta_2+D\sinh\lambda^2\omega_n\theta_2 \end{aligned} \tag{4.3}$$

$$\begin{aligned} g(\theta_2) = {} & -(p_1-p_2)\omega_n A\sinh\omega_n\theta_2 \\ & -(p_1-p_2)\omega_n C\cosh\omega_n\theta_2 \end{aligned} \tag{4.4}$$

and $A$, $B$, $C$, $D$ are constants. The solutions with $C = D = 0$ correspond to an antisymmetrical buckling while the solutions with $A = B = 0$ correspond to a symmetrical buckling. We shall study these two cases separately. Moreover, the case $n = 1$ corresponds to the lowest buckling mode and we shall restrict ourselves to this case only.

## 5. INSTABILITY ASSOCIATED WITH A HYDROSTATIC PRESSURE $p_2$

We take

$$f(\theta_2) = \begin{cases} A\cosh\frac{\pi}{\lambda}\theta_2 + B\cosh\pi\lambda\theta_2 & \text{(bending)} \\ C\sinh\frac{\pi}{\lambda}\theta_2 + D\sinh\pi\lambda\theta_2 & \text{(bulging)} \end{cases} \tag{5.1}$$

$$g(\theta_2) = \begin{cases} -(p_1-p_2)\frac{\pi}{\lambda}A\sinh\frac{\pi}{\lambda}\theta_2 & \text{(bending)} \\ -(p_1-p_2)\frac{\pi}{\lambda}C\cosh\frac{\pi}{\lambda}\theta_2 & \text{(bulging).} \end{cases} \tag{5.2}$$

Substituting (5.1, 5.2) into (4.1), applying the boundary conditions (3.18, 3.19) and setting the determinant equal to zero, we obtain

$$P_1 - P_2 = \frac{4}{\lambda^2} - \frac{8}{1+\lambda^4}\coth\frac{\pi h}{2\lambda^2}\tanh\frac{\pi h}{2} \quad \text{(bending)} \tag{5.3}$$

$$P_1 - P_2 = \frac{4}{\lambda^2} - \frac{8}{1+\lambda^4}\tanh\frac{\pi h}{2\lambda^2}\coth\frac{\pi h}{2} \quad \text{(bulging)} \tag{5.4}$$

where

$$P_1 = p_1/(1+K), \qquad P_2 = p_2/(1+K). \tag{5.5}$$

Equation (2.10) implies that

$$P_1 - P_2 = 2\left(\frac{1}{\lambda^2} - \lambda^2\right). \tag{5.6}$$

It follows from (5.3–5.5) that $\lambda$ satisfies the characteristic equations

$$\frac{(1+\lambda^4)^2}{4\lambda^2} = \coth\frac{\pi h}{2\lambda^2}\tanh\frac{\pi h}{2} \quad \text{(bending)} \tag{5.7}$$

$$\frac{(1+\lambda^4)^2}{4\lambda^2} = \tanh\frac{\pi h}{2\lambda^2}\coth\frac{\pi h}{2} \quad \text{(bulging).} \tag{5.8}$$

These equations were obtained by Biot [8], using a different formulation. Since (5.7, 5.8) do not contain $P_1$ and $P_2$, they are also the characteristic equations for the case $P_2 = 0$. Let $P_1 = P_0$ be the buckling load corresponding to $P_2 = 0$. Then body $B$ is unstable if $P_1$ and $P_2$ satisfy the relation

$$P_1 - P_2 = P_0$$

or

$$\frac{P_1}{P_0}\left(1 - \frac{P_2}{P_1}\right) = 1. \tag{5.9}$$

This equation expresses the fact that the hydrostatic pressure $p_2$ has a stabilizing effect.

## 6. INSTABILITY ASSOCIATED WITH A CONSTANT DIRECTIONAL PRESS $p_2$

Substituting (5.1, 5.2) into (4.1) applying the boundary conditions (3.21, 3.22) and setting the determinant equal to zero, we obtain

$$P_1 - P_2 = \frac{4}{\lambda^2} - \frac{8+4\lambda^2 P_2}{1+\lambda^4+\frac{1}{2}\lambda^2 P_2} \coth\frac{\pi h}{2\lambda^2} \tanh\frac{\pi h}{2} \quad \text{(bending)} \tag{6.1}$$

$$P_1 - P_2 = \frac{4}{\lambda^2} - \frac{8+4\lambda^2 P_2}{1+\lambda^4+\frac{1}{2}\lambda^2 P_2} \tanh\frac{\pi h}{2\lambda^2} \coth\frac{\pi h}{2} \quad \text{(bulging)} \tag{6.2}$$

where $P_1$ and $P_2$ are defined by (5.5). The two pressures $P_1$ and $P_2$ must again satisfy the equation

$$P_1 - P_2 = 2\left(\frac{1}{\lambda^2} - \lambda^2\right). \tag{6.3}$$

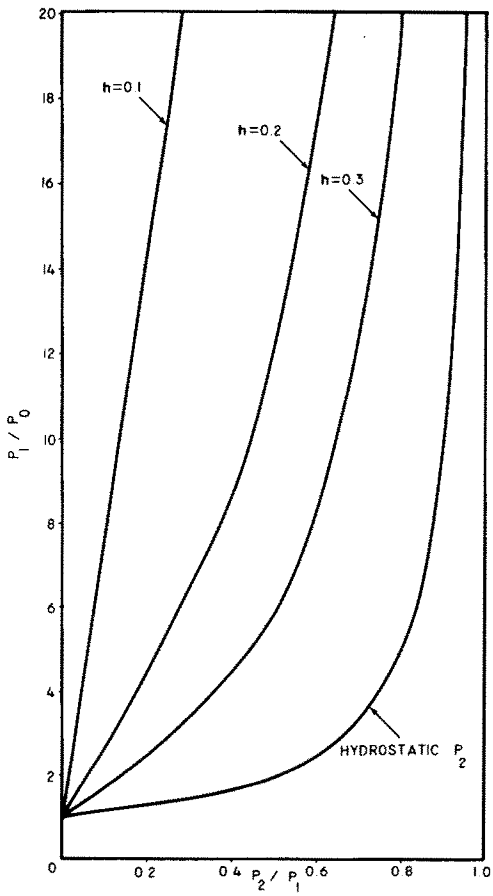


FIG. 2. Antisymmetric instability (bending).

The body $B$ is unstable if $P_1$ and $P_2$ satisfy equation (6.3) and (6.1 or 6.2). It is clear from these equations that no nontrivial solution exists when $P_1 = P_2$.

Equation (6.3) and (6.1 or 6.2) have been solved numerically for different values of $h$ and are plotted in Figs. 2 and 3. It is seen that the constant directional pressure $p_2$ stabilizes the solid for both antisymmetric instability (bending) and symmetric instability (bulging).

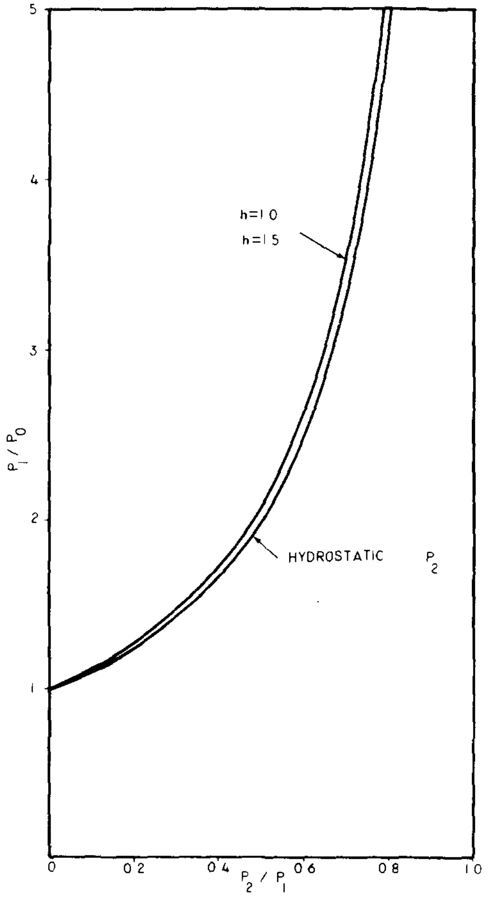


FIG. 3. Symmetric instability (bulging).

## 7. CONCLUSIONS

The results of the present paper are shown in Table 1, which also shows the results obtained by Kerr and Tang. It is seen that when considering a material of the harmonic type which undergoes antisymmetric deformation, a hydrostatic pressure stabilizes the solid while a constant directional pressure destabilizes it. When, however, a Mooney type material is considered, it is found that both types of lateral pressure stabilize the solid for both antisymmetric and symmetric deformations.

TABLE 1

| Lateral pressure $p_2$ | Mooney material (present results) | | Harmonic material (Kerr and Tang) | |
|---|---|---|---|---|
| | Antisymmetric instability (bending) | Symmetric instability (bulging) | Antisymmetric instability (bending) | Symmetric instability (bulging) |
| Hydrostatic pressure | Stabilizing | Stabilizing | Stabilizing | Not treated |
| Constant directional pressure | Stabilizing | Stabilizing | Destabilizing | Not treated |

## REFERENCES


[1] A. D. KERR, On the instability of elastic solids. *Proc. 4th U.S. Nat. Congr. appl. Mech.* 647–656 (1962).
[2] A. D. KERR and S. TANG, The effect of lateral hydrostatic pressure on the instability of elastic solids, particularly beams and plates. *J. appl. Mech.* **33**, 617–622 (1966).
[3] A. D. KERR and S. TANG, The instability of a rectangular solid. *Acta mech.* **IV**, 43–63 (1967).
[4] F. JOHN, Plane strain problems for a perfectly elastic material of harmonic type. *Communs pure appl. Math* **13**, 239–296 (1960).
[5] C. B. SENSENIG, Instability of thick elastic solids. *Communs pure appl. Math.* **17**, 451–491 (1964).
[6] A. E. GREEN and W. ZERNA, *Theoretical Elasticity*. Clarendon Press (1954).
[7] A. E. Green and J. E. ADKINS, *Large Elastic Deformations*. Clarendon Press (1960).
[8] M. A. BIOT, *Mechanics of Incremental Deformations*, Wiley (1965).


(*Received* 30 *September* 1968; *revised* 23 *January* 1969)

**Абстракт**—Исследуется устойчивость прямоугольного твердого тела, в плоском деформационном состоянии, подверженного постоянному осевому и горизонтальному давлению. Рассматриваются два типа горизонтального давления, а именно, гидростатическое давление и постянное, направленное давление. Эти типа задачи исследовались Керром и Тангом для идеально упругого материала гармонического типа. Определяющие уравнения не описывают однако надлежащим образом исвестных материалов, если деформации не оказываются не за слишком большими. Находится, что для случая антисимметрической деформации (изгиб), постоянное направленное давление не обеспечивает устойчивостц твердого тела, тогда как гидростатическое давление обеспечивает его устойчивость. Далее, тело является неустойчивым для случая, когда горизонтальное давление равняется осевому давлению. В настоящей работе рассматривается метериал Мунея. Определяется, что оба типа горизонтального давления обеспечивают устойчивость тела как для антисимметрической так ц для симметриуеской деформации.